Uniform Circular Motion

Kinematics of Circular Motion

An object that moves in a circle at constant speed is said to experience **uniform circular motion**. The magnitude of the velocity remains constant, but the direction of the velocity is constantly changing as the object moves around the circle.

The magnitude of the velocity of the object can be determined as follows:

$$v = \frac{d}{t}$$

For one complete revolution:

$$d = 2\pi r \qquad \text{(circumference)}$$

$$t = T \qquad \text{(period)}$$

$$\boxed{v = \frac{2\pi r}{T}}$$

The direction of the velocity is always **tangent** to the circle (perpendicular to the radius).

Example 1

A 150 g ball at the end of a string is swinging in a horizontal circle of radius 0.60 m. The ball makes exactly 2.00 revolutions in a second. What is the speed of the ball?

Centripetal Acceleration

Any change in the velocity of an object, including a change in direction, is considered to be accelerated motion. Thus, an object experiencing uniform circular motion is continuously accelerating. We call this “centripetal acceleration” because it always points towards the center of the circular path.

The magnitude of the centripetal acceleration is given by:

$$\boxed{a_c = \frac{v^2}{r}}$$

The direction of the centripetal acceleration is always towards the center of the circular path of the object.

Example 2

What is the centripetal acceleration of the ball in Example 1?

Note: The two equations for centripetal motion can also be combined to produce and expression that will allow us to determine centripetal acceleration without first determining the speed.

Substitute $v = \frac{2\pi r}{T}$ into $a_c = \frac{v^2}{r}$.

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r^2}{T^2} \cdot \frac{1}{r} = \frac{4\pi^2 r}{T^2}$$

Thus,

$$\boxed{a_c = \frac{4\pi^2 r}{T^2}}$$

Example 3
The moon's nearly circular orbit about the earth has a radius of about $385000\ km$ and a period of $27.3\ days$. Determine the acceleration of the moon toward the earth.

Homework
Circular Motion Worksheet #1

Circular Motion Worksheet #1

1. A car goes around a curve of radius $50\ m$ at a constant speed of $36\ km/h$. Calculate:
   a) the time required to go around a $90°$ corner. ($7.9\ s$)
   b) the centripetal acceleration of the car. ($2\ m/s^2\ [TTC]$)

2. A car travels in a horizontal circular path of radius $50\ m$. It completes one revolution every $20\ s$. Determine:
   a) frequency. ($0.05\ Hz$)
   b) period. ($20\ s$)
   c) velocity. ($15.7\ m/s$, tangent to the curve)

3. A yoyo is rotating on the end of its string at $10\ rotations/s$ while its owner is demonstrating "walk the dog". If the radius of the yoyo is $3\ cm$, determine:
   a) the speed of a point on the outside edge of the yoyo. ($1.9\ m/s$)
   b) the acceleration of the point. ($118\ m/s^2\ [TTC]$)

4. Determine the radius of a circle in which an airplane traveling at $80\ m/s$ has a centripetal acceleration of $48\ m/s^2$, that is 5 g's! ($133\ m$)

5. At the Six Flags amusement park near Atlanta, the Wheelie carries passengers in a circular path with a radius of $7.7\ m$. The ride makes a complete rotation every $4\ s$. What acceleration at the top of the ride and the acceleration at the bottom of the ride. ($19\ m/s^2\ [down]$ at top $[up]$ at bottom)

6. Imagine a giant donut shaped space station located so far from all heavenly bodies that the force of gravity may be neglected. To enable the occupants to live a "normal" life, the donut rotates and inhabitants live on the part of the donut farthest from the center. If the outside diameter of the space station is $1.5\ km$, what must be its period of rotation so that the passengers at the periphery will perceive an artificial gravity equal to the normal gravity at the earth's surface? **Hint**: a_c must be $9.8\ m/s^2$ ($55\ s$)

7. The pilot of an airplane, which has been diving at a speed of $540\ km/h$, pulls out of the dive at constant speed. What is the minimum radius of the plane's circular path in order that the acceleration of the pilot at the lowest point will not exceed 7 g? **Hint**: convert 7 g's to an acceleration in m/s^2 ($328\ m$)

8. A jet plane traveling $500\ m/s$ pulls out of a dive by moving in an arc of radius $4\ km$. What is the plane's acceleration in g's? (6.38 g's)

   Note: $1\ g = 9.8\ m/s^2$